APPENDIX A

Proof of Proposition 1:

From (14), we have $\hat{c}+(B-1)\int_0^{\hat{c}}(\hat{c}-c)dH(c)=(B/\varepsilon)(1-f)(1-1/N)$. We first prove $\hat{c}$ is well-defined. The derivative with respect to $\hat{c}$ of the left-hand side (LHS) of (14) is $1+(B-1)H(\hat{c})$, which is positive. Therefore, the LHS is strictly increasing in $\hat{c}$, while the right-hand side (RHS) of (14) is independent of $\hat{c}$. Moreover, the LHS converges to zero as $\hat{c}$ tends to zero. We infer the threshold $\hat{c}$ exists.

The RHS of (14) is increasing in *N* and decreasing in ε and *f*. Since the LHS of (14) is increasing in $\hat{c}$, we deduce $\hat{c}$ is increasing in *N* and decreasing in ε and *f*, thus the time to standardize $T \equiv H(\hat{c})^{-1}$ is decreasing in *N* and increasing in ε and *f*.

Take the total derivative of (14) with respect to *B*, to obtain

$$(d\hat{c}/dB)B[1+(B-1)H(\hat{c})]=(B/\varepsilon)(1-f)(1-1/N)-B\int_0^{\hat{c}}(\hat{c}-c)dH(c). \qquad \text{(a3)}$$

From (14), the RHS of (a3) equals $\hat{c}[1-H(\hat{c})]+\int_0^{\hat{c}}cdH(c)$, which is positive. Hence, the threshold $\hat{c}$ is increasing in *B*, which in turn is increasing in the growth rate of consumer income *g* and the discount factor β. We infer that $T \equiv H(\hat{c})^{-1}$ is decreasing in *g* and β.

Proof of Proposition 2:

The definition of $\hat{c}$, given by (14), is identical to the definition of $\hat{c}_{SP}$, given by (23), except for the right-hand side (RHS), which is $(B/\varepsilon)(1-f)(1-1/N)$ for $\hat{c}$ while it is $(1-1/N)B/(\varepsilon-1)$ for $\hat{c}_{SP}$. The RHS is greater for the definition of $\hat{c}_{SP}$ compared to the definition of $\hat{c}$. We showed in the proof of Proposition 1 that the left-hand side (LHS) of (14) is increasing in $\hat{c}$, and similarly the LHS of (23) is increasing in $\hat{c}_{SP}$, so we infer $\hat{c}_{SP} > \hat{c}$, which implies $H(\hat{c}_{SP})^{-1} < H(\hat{c})^{-1}$.

Proof of Proposition 4:

The number of monopolists and standards *N* is made endogenous by allowing for costly entry into the industry. Let *X* denote the cost to a monopolist of entering the industry at birth (with a proprietary standard) per dollar of consumer income Y_t. Once a monopolist is active, it may become compatible by incurring the coordination cost to resolve the standards war. The following lemma proves that the expected value of a monopolist per dollar of consumer income *EV* is decreasing in *N*.

LEMMA A1: *Suppose (A1) and (A2) hold. The expected value of a monopolist per dollar of consumer income EV is decreasing in the number of monopolists and standards N.*

Proof: From (13), we have $EV = (B/\varepsilon)(1-(1-1/N)f) - \hat{c} + \int_0^{\hat{c}} (\hat{c}-c)dH(c)$. Taking its total derivative with respect to *N*, we obtain $\dfrac{dEV}{dN} = -\dfrac{Bf}{\varepsilon N^2} - [1-H(\hat{c})]\dfrac{d\hat{c}}{dN}$. We showed in the proof of Proposition 1 that $\hat{c}$ is increasing in *N*, so *EV* is decreasing in *N*.

By virtue of Lemma A1, the more monopolists enter the industry, the lower is the expected value of a monopolist (per dollar of consumer income) *EV*. It follows that monopolists enter up to the point that the cost of entry *X* equals the expected present value of being active. Because the initial consumer income level is $Y_0 = 1$, in equilibrium we have $EV = X$, which determines the number of monopolists and standards.

Setting $EV = X$ in (13), it becomes $X = (B/\varepsilon)(1-(1-1/N_E)f) - \hat{c}_E + \int_0^{\hat{c}_E} (\hat{c}_E - c)dH(c)$. From (14), we have $\hat{c}_E + (B-1)\int_0^{\hat{c}_E} (\hat{c}_E - c)dH(c) = (B/\varepsilon)(1-f)(1-1/N_E)$. This yields two equations in the two unknowns, $\hat{c}_E$ and N_E. The threshold coordination cost is given by

$$\hat{c}_E + (Bf-1)\int_0^{\hat{c}_E} (\hat{c}_E - c)dH(c) = (1-f)(B/\varepsilon - X). \tag{a1}$$

Given the definition of $\hat{c}_E$ in (a1), the equilibrium number of monopolists and standards is

$$N_E = (1-f)(B/\varepsilon)\left[(1-f)(B/\varepsilon) - \hat{c}_E - (B-1)\int_0^{\hat{c}_E} (\hat{c}_E - c)dH(c)\right]^{-1}. \tag{a2}$$

For the threshold $\hat{c}_E$ to exist, we must assume

$$B/\varepsilon > X. \tag{A3}$$

If $B/\varepsilon \le X$, then the industry is never born because no entry occurs. The condition $B/\varepsilon > X$ is intuitive: if the entry cost *X* is greater than the expected present value of being compatible B/ε, then no monopolist has an incentive to enter.

The following proposition proves the threshold $\hat{c}_E$ is well-defined and describes the properties of the time to standardize $T_E \equiv H(\hat{c}_E)^{-1}$.

PROPOSITION A1: *Suppose (A1)-(A3) hold. The time to standardize T_E is decreasing in the growth rate g and the discount factor β; and increasing in the price elasticity of demand ε, the entry cost X, and the compatibility cost f.*

Proof of Proposition A1: From (a1), we have $\hat{c}_E + (Bf-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c) = (1-f)(B/\varepsilon - X)$. We first prove $\hat{c}_E$ is well-defined. The derivative with respect to $\hat{c}_E$ of the left-hand side (LHS) of (a1) is $1+(Bf-1)H(\hat{c}_E)$, which is positive irrespective of the sign of $Bf-1$. Therefore, the LHS is increasing in $\hat{c}_E$, while the right-hand side (RHS) of (30) is independent of $\hat{c}_E$. Moreover, the LHS converges to zero as $\hat{c}_E$ tends to zero. We infer the threshold $\hat{c}_E$ exists if $B/\varepsilon > X$, which is assumption (A3).

The right-hand side (RHS) of (a1) is decreasing in the price elasticity of demand ε and the entry cost *X*. Since the LHS of (a1) is increasing in $\hat{c}_E$, and the time to standardize $T_E \equiv H(\hat{c}_E)^{-1}$ is decreasing in $\hat{c}_E$, we infer the time to standardize is increasing in ε and *X*.

Take the total derivative of (a1) with respect to *B*, to obtain

$$(d\hat{c}_E/dB)[1+(Bf-1)H(\hat{c}_E)] = (1-f)/\varepsilon - f\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c). \qquad \text{(a4)}$$

Using (a1), (a4) becomes

$$(d\hat{c}_E/dB)B[1+(Bf-1)H(\hat{c}_E)] = (1-f)X + \hat{c}_E[1-H(\hat{c}_E)] + \int_0^{\hat{c}_E} c\,dH(c). \qquad \text{(a5)}$$

The RHS of (a5) is positive, so $d\hat{c}_E/dB > 0$. Since *B* is increasing in the growth rate *g* and the discount factor β, and $T_E \equiv H(\hat{c}_E)^{-1}$ is decreasing in $\hat{c}_E$, we infer that the time to standardize is decreasing in *g* and β.

Take the total derivative of (a1) with respect to *f*, to obtain

$$(d\hat{c}_E / df)[1+(Bf-1)H(\hat{c}_E)] = -(B/\varepsilon - X) - B\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c). \tag{a6}$$

The RHS of (a6) is negative, so $d\hat{c}_E / df < 0$. Since $T_E \equiv H(\hat{c}_E)^{-1}$ is decreasing in $\hat{c}_E$, we infer that the time to standardize is increasing in *f*.

The following proposition describes the properties of the equilibrium number of monopolists and standards.

PROPOSITION A2: *Suppose (A1)-(A3) hold. The equilibrium number of monopolists and standards* N_E *is increasing in the growth rate g and the discount factor* β*; and decreasing in the price elasticity of demand* ε*, the entry cost X, and the compatibility cost f.*

Proof of Proposition A2: From (a2), we have $N_E = (1-f)(B/\varepsilon)\left[(1-f)(B/\varepsilon) - \hat{c}_E - (B-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c)\right]^{-1}$. This is increasing in $\hat{c}_E$ since the derivative of $\hat{c}_E + (B-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c)$ with respect to $\hat{c}_E$ is $1+(B-1)H(\hat{c}_E)$ and N_E is increasing in $\hat{c}_E + (B-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c)$. We showed in the proof of Proposition 4 that $\hat{c}_E$ is decreasing in the entry cost *X*, so N_E is also decreasing in *X*.

Take the total derivative of (31) with respect to *f*:

$$dN_E / df = (B/\varepsilon)\frac{\hat{c}_E + (B-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c) + (1-f)[1+(B-1)H(\hat{c}_E)]d\hat{c}_E / df}{\left[(1-f)(B/\varepsilon) - \hat{c}_E - (B-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c)\right]^2}. \tag{a7}$$

Using the expression for $d\hat{c}_E / df$ from (a6), together with (a2), after extensive algebra, we can show that $dN_E / df < 0$.

Take the total derivative of (a2) with respect to *B*:

$$dN_E / dB = ((1-f)/\varepsilon)\frac{B[1+(B-1)H(\hat{c}_E)]d\hat{c}_E / dB - \hat{c}_E + \int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c)}{\left[(1-f)(B/\varepsilon) - \hat{c}_E - (B-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c)\right]^2}. \tag{a8}$$

Using the expression for $d\hat{c}_E / dB$ from (a5), together with (a2), after extensive algebra, we can show that $dN_E / dB > 0$. Since *B* is increasing in the growth rate *g* and the discount factor β, we infer that the number of monopolists and standards is increasing in *g* and β.

We may express (a2) as follows:

$$N_E = (1-f)B\left[(1-f)B - \varepsilon\left(\hat{c}_E + (B-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c)\right)\right]^{-1}. \tag{a9}$$

Therefore, N_E is increasing in $\Omega \equiv \varepsilon\left(\hat{c}_E + (B-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c)\right)$, so the sign of $dN_E / d\varepsilon$ equals the sign of $d\Omega / d\varepsilon$, which is given by:

$$d\Omega / d\varepsilon = \hat{c}_E + (B-1)\int_0^{\hat{c}_E}(\hat{c}_E - c)dH(c) + \varepsilon[1 + (B-1)H(\hat{c}_E)]d\hat{c}_E / d\varepsilon. \tag{a10}$$

Take the total derivative of (a1) with respect to ε:

$$(d\hat{c}_E / d\varepsilon)[1 - (1 - Bf)H(\hat{c}_E)] = -(1-f)B / \varepsilon^2. \tag{a11}$$

Applying (a11) to (a10), together with (a1) and (a2), after extensive algebra, we can show that $d\Omega / d\varepsilon < 0$, implying that $dN_E / d\varepsilon < 0$.

Entry into the industry is a function of its profitability and the cost of entry. The industry is more profitable the smaller is the interest rate, the greater is the growth rate of consumer income, the smaller is the compatibility cost, and the less elastic is demand (due to the standard monopoly markup over marginal cost). The more profitable is the industry and the smaller is the entry cost, the more entry occurs; and because each monopolist has a proprietary standard, the more standards are established when the industry is born.

We proved in Proposition 1 that, when the number of monopolists and standards *N* is exogenous, the time to standardize *T* is decreasing in *N*, the growth rate *g*, and the discount factor β; and increasing in the elasticity ε and the compatibility cost *f*. Proposition A2 showed that the equilibrium number of monopolists and standards N_E is increasing in *g* and β; and decreasing in ε, the entry cost *X*, and *f*. It follows that there are two reasons why the time to standardize T_E is increasing in ε and *f* and decreasing in *g* and β: first, due to their direct effect on *T*; and second, due to their indirect effect on *T* operating via the equilibrium number of monopolists and standards N_E. Finally, the time to standardize T_E is increasing in the entry cost *X* since *T* is decreasing in *N* and N_E is decreasing in *X*.

By comparing the properties of the time to standardize T_E with those of the number of monopolists and standards N_E listed in Propositions A1 and A2, respectively, we can prove Proposition 4